

Griffiths

$$6.7(a). \quad (m_1 m_2 c^2)^2 = m_1^2 c^2 m_2^2 c^2$$

$$m_1^2 c^2 = \left(\frac{E_1}{c}\right)^2 - |\mathbf{p}|^2, \quad m_2^2 c^2 = \left(\frac{E_2}{c}\right)^2 - |\mathbf{p}|^2$$

$$\Rightarrow (m_1 m_2 c^2)^2 = \left(\frac{E_1}{c}\right)^2 \left(\frac{E_2}{c}\right)^2 - \left(\frac{E_1}{c}\right)^2 |\mathbf{p}|^2 - \left(\frac{E_2}{c}\right)^2 |\mathbf{p}|^2 + |\mathbf{p}|^4$$

$$(\mathbf{p}_1 \cdot \mathbf{p}_2)^2 = \left[\left(\frac{E_1}{c}\right) \left(\frac{E_2}{c}\right) + |\mathbf{p}|^2 \right]^2$$

$$= \left(\frac{E_1}{c}\right)^2 \left(\frac{E_2}{c}\right)^2 + 2 \left(\frac{E_1}{c}\right) \left(\frac{E_2}{c}\right) |\mathbf{p}|^2 + |\mathbf{p}|^4$$

$$(\mathbf{p}_1 \cdot \mathbf{p}_2)^2 - (m_1 m_2 c^2)^2 = 2 \left(\frac{E_1}{c}\right) \left(\frac{E_2}{c}\right) |\mathbf{p}|^2 + \left(\frac{E_1}{c}\right)^2 |\mathbf{p}|^2 + \left(\frac{E_2}{c}\right)^2 |\mathbf{p}|^2$$

$$= \left[\left(\frac{E_1}{c}\right) |\mathbf{p}| + \left(\frac{E_2}{c}\right) |\mathbf{p}| \right]^2$$

$$\sqrt{(\mathbf{p}_1 \cdot \mathbf{p}_2)^2 - (m_1 m_2 c^2)^2} = \frac{E_1}{c} |\mathbf{p}| + \frac{E_2}{c} |\mathbf{p}| = \boxed{\frac{(E_1 + E_2) |\mathbf{p}|}{c}}$$

(b). for particle 2 at rest, $\mathbf{p}_1 = \left(\frac{E_1}{c}, \vec{p}\right)$, $\mathbf{p}_2 = (m_b c, 0)$

$$\mathbf{p}_1 \cdot \mathbf{p}_2 = \frac{E_1}{c} m_b c, \quad (\mathbf{p}_1 \cdot \mathbf{p}_2)^2 - m_a^2 m_b^2 c^4 = E_1^2 m_b^2 - m_a^2 m_b^2 c^4$$

$$= m_b^2 [E_1^2 - m_a^2 c^4]$$

$$= m_b^2 |\mathbf{p}|^2 c^2$$

$$\Rightarrow \sqrt{(\mathbf{p}_1 \cdot \mathbf{p}_2)^2 - m_a^2 m_b^2 c^4} = \boxed{m_b |\mathbf{p}| c}$$